# Digital to Analog Converter

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# Topics of Discussion

- What is a DAC?
- Types of DAC Circuits
  - Resistor-string DAC
  - N-Bit Binary weighted DAC
  - R-2R Ladder DAC
- Specifications of DAC
- Applications

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# What is a DAC?

- *Digital-to-Analog Converter*: An electronic device, *often* an integrated circuit, that converts a digital number into a corresponding analog voltage or current.
- Relation between analog signal and digital equivalent

$$V_a = \sum_{i=1}^{n} b_i 2^{-i} \times V_{ref}$$

- AD conversion – $V_a$ -> $b_i$ (encoder) ex) Transducer interface
- DA conversion – $b_i$ -> $V_a$ (decoder) ex) motor, heater control



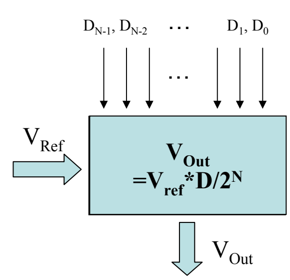
# DAC configurations

- Assume the analog signal is a voltage

N-bit digital word

⇩

Analog voltage signal

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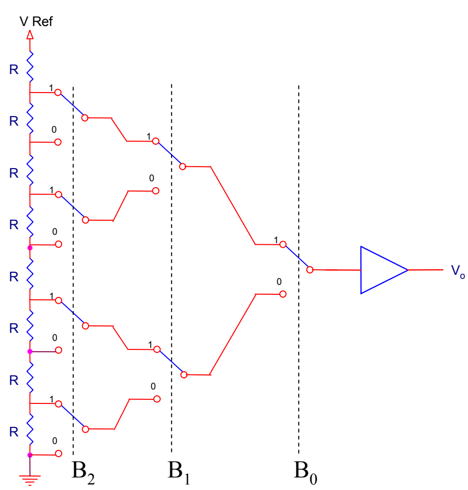
# Types of DAC Circuits

1. Resistor-string
2. N-Bit Binary Weighted Resistor
3. R-2R Ladder



# A Resistor-string DAC




Example of 3 bit resistor-string DAC

Required component

- a resistor string
- a set of switches – select output to use
- opamp – buffer

$$V_0 = V_{Ref} \sum_{i=0}^{N-1} \frac{b_i}{2^{i+1}}$$

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# Resistor String DAC Example

- How many resistors and switches would be required to implement an 8 bit resistor-string DAC?

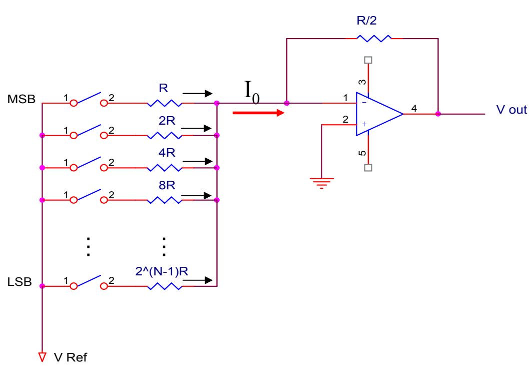
Ans) # of resistors = $2^N = 2^8 = 256$

$$\text{\# of switches} = \sum_{i=0}^{N-1} 2^i = 2^8 - 1 = 255$$

Impractical for converters with more than a few bits of resolution



# N-Bit Binary Weighted Resistor




$$I_0 = V_R \sum_{i=1}^{N} \frac{b_i}{2^{(i-1)} R}$$

$$V_0 = -R_f I_0$$

# N-bit binary weighted Example

- Find output voltage, current, and resolution for a binary weighted resistor DAC of 4 bits

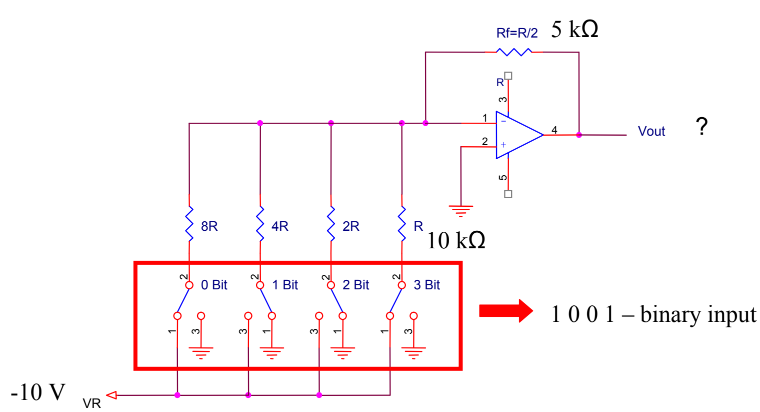
  - given condition

  $R = 10\ k\Omega$, $R_f = 5\ k\Omega$, $V_R = -10\ V$

  Applied binary word is 1001



# Example Solutions



# Example Solutions (Cont.)

Ans)

$$I_0 = \frac{-10\ \ \mathrm{V}}{\Omega}\left[\frac{1}{2^0\times10^4}+\frac{0}{2^1\times10^4}+\frac{0}{2^2\times10^4}+\frac{1}{2^3\times10^4}\right]$$

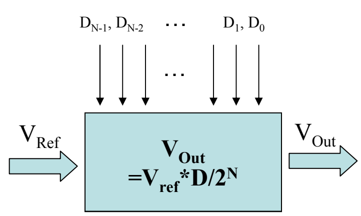
$$= \ -0.001125\ \mathrm{A}$$

$$V_0 = -\,R_f I_0 = -(5\times10^3\,\Omega)\times(-0.001125\mathrm{A}) = 5.625\mathrm{V}$$

$$\text{Resolution} = \text{LSB} = \frac{V_{Ref}}{2^n} = \frac{10\mathrm{V}}{2^4} = 0.625\mathrm{V}$$



# Example Solutions (Cont.)




- Binary Word input = $1001_2 = 9_{10}$
- # of input combination : $2^4 = 16$
- $V_{out} = V_{ref} * D/2^N = 10*9/16 = 5.625$ V

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# Limitations of the Binary Weighted DAC

1. If R = 10 kΩ, 8 bits DAC, and $V_{Ref}$ = 10 V
   $R_8 = 2^{8-1}*(10\ k\Omega) = 1280\ k\Omega$
   $I_8 = V_{Ref}/R_8 = 10V/1280\ k\Omega = 7.8\ \mu A$

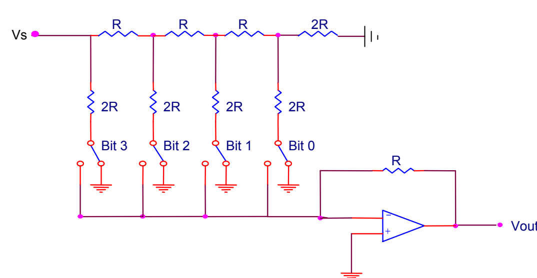
**Op-amps that can handle those currents are rare and expensive.**

2. If R = 10 Ω and $V_{Ref}$ = 10 V
   $R_1 = 2^{1-1}*(10\ \Omega) = 10\ \Omega$
   $I_1 = V_{Ref}/R_1 = 10V/10\ \Omega = 1\ A$

**This current is more than a typical op-amp can handle.**



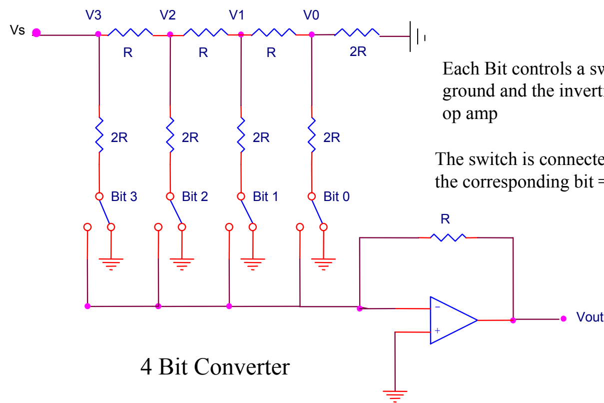
# R-2R Resistor Ladder DAC




•Simplest type of DAC

•Requires only two precision resistance value (R and 2R)

# R-2R Resistor Ladder DAC



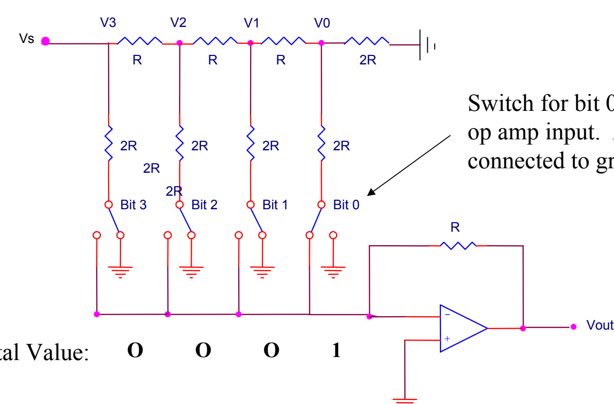

Each Bit controls a switch between ground and the inverting input of the op amp

The switch is connected to ground if the corresponding bit = 0

4 Bit Converter

# R-2R DAC Example

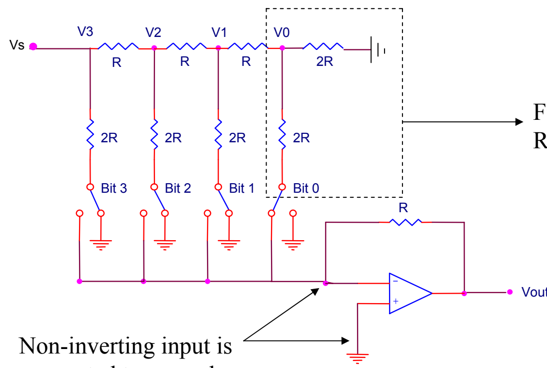
Ex. Convert 0001 to analog




Switch for bit 0 is connected to op amp input. All other switches connected to ground

Digital Value: 0 0 0 1

# R-2R DAC Example (cont.)

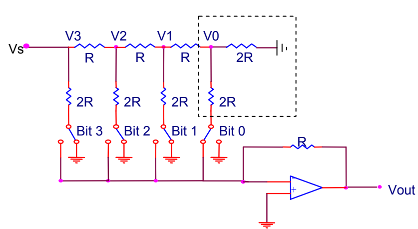


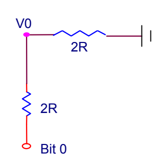

Find Equivalent Resistance

Non-inverting input is connected to ground. Therefore, inverting input is at virtual ground



# R-2R DAC Example (cont.)



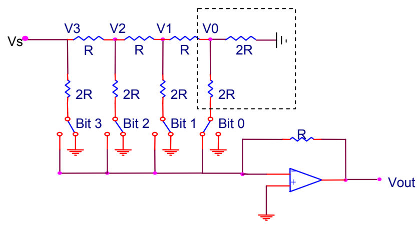




Req = 2R // 2R = R

Apply voltage division:

$$V_0 = V_1 \times \frac{Req}{(Req+R)} \longrightarrow \boxed{V_0 = \tfrac{1}{2} V_1}$$

# R-2R DAC Example (cont.)




We have shown that $V_0 = ½ V_1$

Similarly, It can be proven that :

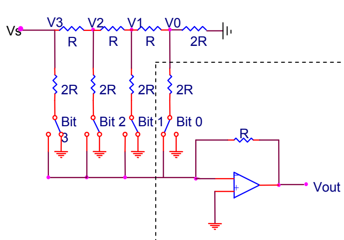
$V_1 = ½ V_2$ AND $V_2 = ½ V_3$

Therefore:

$V_0 = 1/8\ V_3 = 1/8\ V_s$



# R-2R DAC Example (cont.)




$V_0$ is the input to the inverting amplifier circuit, which has a gain of:

Av = -R/2R = -1/2

Therefore, the analog output voltage corresponding to the binary input 0001 is:

$Vout_0 = Av\ (V_0)$

$= (-1/2)(1/8*Vs)$

$Vout_0 = -1/16\ Vs$

# R-2R DAC Example (cont.)

We have shown that the analog output voltage for the digital input 0001 is:

$V_{out0}$ = -1/16 Vs

Similarly, it can be show that:

For input = 0010: $V_{out1}$ = -1/8 Vs

For input = 0100: $V_{out2}$ = -1/4 Vs

For input = 1000: $V_{out3}$ = -1/2 Vs

The output for any combination of bits comprising the input binary number can now be found using the principle of superposition:

$$V_{out} = b_3V_{out3} + b_2V_{out2} + b_1V_{out1} + b_0V_{out0}$$



# General DAC Characteristics

There are six key parameters you should consider when choosing a DAC.

- Reference Voltage
- Resolution
- Linearity
- Speed
- Settling Time
- Error

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# Reference Voltage

To a large extent, the characteristics of a DAC are defined by its reference voltage.

- Non-multiplier DAC: $V_{ref}$ is fixed (specified by the manufacturer)
- Multiplier DAC: $V_{ref}$ is provided via an external source



# Full Scale Voltage

Defined as the output when the digital input word is all 1's.

$$V_{fs} = V_{ref}\left(\frac{2^N - 1}{2^N}\right)$$

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# Resolution

Resolution is a measure of precision, not accuracy. It is defined as the voltage change corresponding to changing the LSB.

- Many options in the 8-16 bit range, with 12 bits being a typical cost / resolution trade off.
- More bits → More steps → Greater Resolution

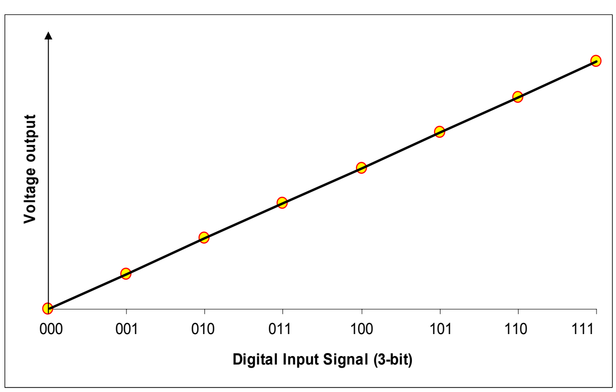
$$\text{Resolution} = V_{LSB} = V_{Ref} / 2^N$$

*where N is the number of bits



# Linearity

- Ideally, a DAC will produce a linear relationship between a binary word and analog output

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# Speed

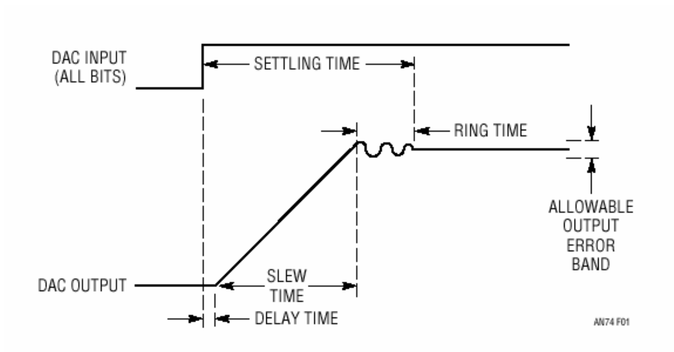
Usually specified as conversion or sampling rate.

- High speed DACs are typically defined as >1MS/s (1Mhz)
- Some current 12-16 bit DACs can reach the 1GHz range
- Conversion of input signal is limited by
  - Clock speed of the input signal
  - Settling time of the DAC

# Settling Time

- Ideally, an instantaneous change in analog voltage would occur when a new binary word enters into a DAC.
- Settling time is the time taken by the DAC to reach ½ of the LSB of its new voltage.
- Components include delay, slew time, and ring time.
- Fast converters reduce slew time, but usually result in longer ring times.
- Delay time is normally a small term.

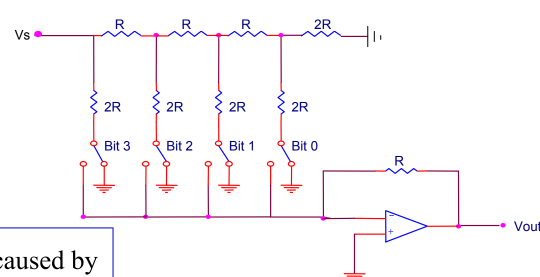
# Settling Time (cont.)






## Cause of Delay Time, Settling Time, and Ring Time

R-2R Resistor Ladder DAC




Delay time is caused by the time it takes to change these switches based on input bits.

Settling time and Ring time is determined by the op-amp's slew rate

# Possible Error

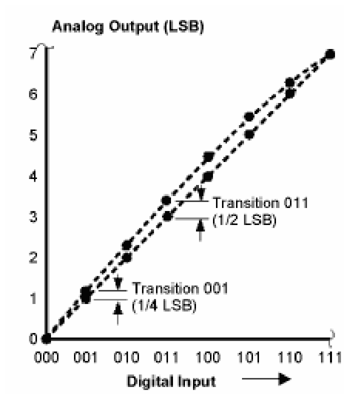
Because we do not live in an "ideal" world, considerations for possible error should be made.

- Non-Linearity
  - Integral
  - Differential
- Non Monotonicity
- Offset Error
- Gain Error



# Integral Non-linearity

- Defined as the deviation of a DAC's transfer function from a straight line.
- The straight line can be a best approximation to the actual transfer function, or a line drawn between the transfer function's end points (after subtracting the gain and offset errors

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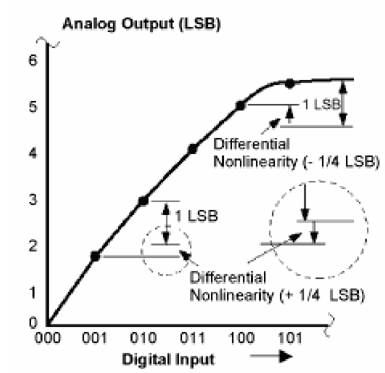
# Differential Non-Linearity

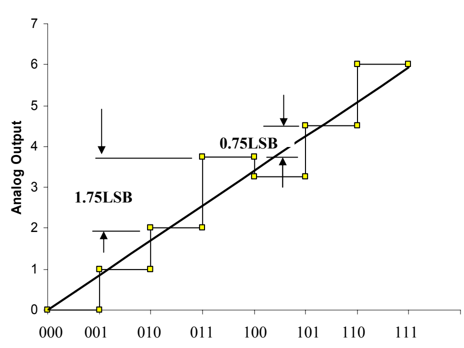
- The difference between an actual step height and the ideal value of 1LSB.
- Should be less than or equal to one to insure monotonicity.






# Non-Monotonicity

- A monotonic DAC yields an increase in output as input increases.
- If a differential non-linearity of greater than 1LSB occurs, increasing the digital input may actually result in a decreased output.

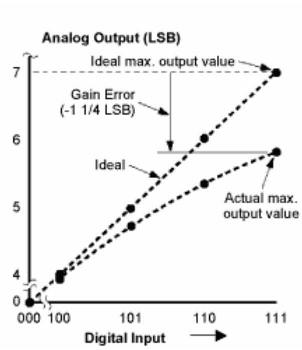


# Gain Error

•Defined as the difference between the ideal max output voltage and the actual max output voltage (after subtracting offset error).

•Changes the slope of the output, thereby creating the same percentage error for each step.

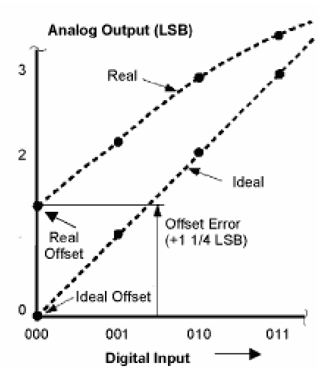
•Expressed in mV as a percentage of the maximum output.





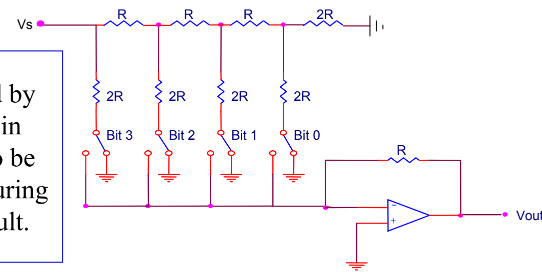

# Offset Error

- The offset error equals the analog output when the digital signal is zero.
- Typically defined in absolute millivolts with (10mV being acceptable).



Cause of Non-Linearity, Gain error, and Offset error

R-2R Resistor Ladder DAC




Non-linearity is caused by the resistors. Resistors in D/A converters need to be matched but manufacturing exact resistors is difficult.

Gain Error and Offset Error is caused by op-amp and or op-amp's feedback resistor



# DAC Applications

- DAC's can be found in any device that interfac es digital and analog circuitry
- Analog Displays
- Digital Control Systems
- Digital Audio
- Communications
- Countless other applications

# References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# Questions ??